EFFECT OF DISCRETE STRUCTURE OF "CONTINUOUS" PHASE OF HEAT TRANSFER IN A FLUIDIZED BED

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The heating of a semi-infinite layer of particles adjacent to a heated surface is investigated analytically in a one-dimensional approximation. The conditions in which such a system can be regarded as a homogeneous isotropic medium are found.

The heat from the surface of a body immersed in a fluidized bed of finely granular material is given up to packets of particles in contact with the surface. These particles are periodically dislodged by ascending gas bubbles and replaced by new ones. Mickley and Fairbanks [1], who were the first to present an analytical description of this process, regarded the packet as a semi-infinite homogeneous mass with a surface temperature t_w, which remained constant during the entire time of heating of the packet. Accordingly, the instantaneous (at time τ) value of the coefficient of heat transfer between the body and the fluidized bed is

$$\alpha_\tau = R_\lambda^{-1} = (\lambda_{c.p} c_m \rho_{c.p})^{1/2} (\pi\tau)^{-1/2},$$

and the mean coefficient during the time τ_c of contact of the packet with the surface is

$$\alpha' = 0.5\, R_c^{-1} = 0.5 (\lambda_{c.p}\, c_m \rho_{c.p})^{1/2} (\pi\tau_c)^{-1/2}.$$

It follows from these expressions that the heat transfer coefficients α and α^1 tend asymptotically to infinity when $\tau \to 0$. Yet it is well known [2, 3] that when the time of contact is reduced to a certain value the heat transfer coefficient ceases to increase and tends towards a limiting (ultimate) value, which depends on the thermal resistance between the surface of the body and the first row of particles in contact with it. To eliminate this contradiction we [4] introduced a "contact thermal resistance" R_{con}. Then

$$\alpha_\tau = \frac{1}{R_{con}} \exp\left[\left(\frac{R_\lambda}{R_{con}}\right)^2 \frac{1}{\pi}\right] \operatorname{erfc}\left(\frac{R_\lambda}{R_{con}} \frac{1}{\sqrt{\pi}}\right). \quad (1)$$

The mean value α' in time τ_c can be found by integration by parts:

$$\alpha' = \frac{1}{\tau_c} \int_0^{\tau_c} \alpha_\tau \, d\tau = \frac{A}{R_c}, \quad (2)$$

$$A = \frac{1}{y^2} (\exp y^2 \operatorname{erfc} y - 1) + 2; \quad y = \frac{R_c}{R_{con} \sqrt{\pi}}. \quad (2')$$

The function $\exp y^2 \operatorname{erfc} y$ is tabulated in [5]. The values of the coefficient A are given in Table 1.

To simplify the calculations we suggested [4] that the solution (1) should be replaced by the simpler approximate formula

$$\alpha_\tau = \beta (R_{con} + R_\lambda)^{-1}.$$

Calculations showed that the correction factor β does not exceed 1.20. Gel'perin, Ainshtein, and Zaikovskii drew attention to the fact that the mean values of the heat transfer coefficient can be calculated in a similar way: $\alpha' = (R_{con} + 0.5\, R_c)^{-1}$. The deviation of the values calculated from this formula from the values given by (2), (2') does not exceed ±5%, so that no correction is necessary. If one considers that for part of the time f_0 the heat transfer surface is not in contact with the particles, but with a gas bubble (at this instant heat transfer is negligible), then, if the mean time of contact of the packet $\tau_c = (1 - f_0)\varphi_t^{-1}$ is known, it is easy to calculate the coefficient of heat transfer from the fluidized bed to the submerged body:

$$\alpha = (1 - f_0) \frac{A}{R_c} \simeq \frac{1 - f_0}{(0.5 + R_{con}/R_c) R_c}. \quad (3)$$

Calculation from (3) gives quite good agreement with experiment for a fluidized bed of small particles, but for a bed of larger particles ($d \ge 0.3$ mm) there are deviations [4]. These deviations are due to the fact that during the time of contact of the packet with the surface (usually less than 0.5–1 sec) only a thin layer of the material, with a thickness of only a few particle diameters, is heated (or cooled). With such conditions a discrete medium composed of individual particles and interstitial gas cannot be regarded as homogeneous.

We will determine the conditions in which the packet of particles in contact with the surface can be regarded as a homogeneous isotropic medium. To do this we consider an equivalent system (see Fig. 1, b) in which the medium consists of layers of thickness d separated from one another (thermally) by contact thermal resistances r_{con}, which are assumed to be independent of time. The thermal conductivity of the layers is assumed to be infinitely high and the product of their density and heat capacity is assumed to be equal to that of the continuous phase of the fluidized bed. During the heating of any i-th layer it obtains heat from the (i − 1)th layer and gives it up to the (i + 1)th layer. Hence,

$$\frac{t_{i-1} - t_i}{r_{con}} - \frac{t_i - t_{i+1}}{r_{con}} = c_m \rho_{c.p}\, d \frac{dt_i}{d\tau}. \quad (4)$$

We introduce

$$\Theta_i = \frac{t_i - t_0}{t_w - t_0}, \ (\Theta_w \equiv 1),$$

$$x = \frac{2\tau}{c_m \rho_{c.p}\, d r_{con}} = \frac{2\tau\lambda_{c.p}}{c_m \rho_{c.p} d^2} = 2\text{Fo}. \quad (4')$$

In the last relationship we have used $\lambda_{c.p} = d/r_{con}$.

Then

$$\Theta_i = \frac{1}{2}\exp(-x)\int_0^x \exp(x)(\Theta_{i-1} + \Theta_{i+1})\,dx. \qquad (5)$$

The solution of a similar problem is known [5]. In the symbols used here the temperature of the first row of particles is given by the expression

$$\Theta_1 = \int_0^x \exp(-x) J_1(x)\frac{dx}{x} = $$

$$= \int_0^1 \exp(-xz) z^{-1} J_1(xz)\,dz,$$

where $J_1(x)$ is a Bessel function of imaginary argument of the first kind and first order. Using tables of integrals ([7], p. 728) we can easily find

$$\Theta_1 = \frac{x}{2}\ {}_2F_2\left(\frac{3}{2},\ 1;\ 3,\ 2;\ -2x\right).$$

Expanding the generalized hypergeometric series ${}_2F_2$ ([7], p. 1059) we can finally write

$$\Theta_1 = \frac{x}{1/2!}\sum_{k=0}^{\infty}\frac{(k+1/2)!}{(k+1)!\,(k+2)!}(-2x)^k. \qquad (5')$$

This method of solution is suitable only in the case where the thermal resistances between all the layers are the same and equal to the thermal resistance between the plate and the first layer. In addition, this solution is difficult to visualize. Hence, we find the temperature of the first row of particles by the method of successive approximations. For the first approximation we take $\Theta_2^{\mathrm{I}} = 0$. Then, since $\Theta_{i-1}^{\mathrm{I}} = \Theta_W = 1$, we obtain from (5)

$$\Theta_1 = \frac{1}{2}[1 - \exp(-x)]. \qquad (6)$$

In the second approximation $\Theta_3^{\mathrm{II}} = 0$. From (5) we find

$$\Theta_2^{\mathrm{II}} = \frac{1}{2}\exp(-x)\int_0^x \exp(x)(\Theta_1^{\mathrm{I}} + \Theta_3^{\mathrm{II}})\,dx =$$

$$= \frac{1}{2^2}[1 - \exp(-x) - \exp(-x)\,x].$$

From Θ_2^{II} and $\Theta_W = 1$ we find Θ_1^{III} from (5), and so on.

Table 1

Values of Coefficient A from Formula (2')

$y = \frac{R_c}{R_{con}\sqrt{\pi}}$	0.1	0.25	0.4	0.55	0.65	0.8	0.95
A	0.177	0.355	0.53	0.68	0.765	0.863	0.955
y	1.2	1.5	1.8	2.1	2.4	3.0	∞
A	1.09	1.19	1.28	1.35	1.42	1.51	2

In the fourth approximation, for instance,

$$\Theta_1^{\mathrm{IV}} = \frac{1}{2}[1 - \exp(-x)] +$$

$$+ \frac{1}{8}\left[1 - \exp(-x)\left(1 + \frac{x}{1!} + \frac{x^2}{2!}\right)\right] +$$

$$+ \frac{1}{16}\left[1 - \exp(-x)\left(1 + \frac{x}{1!} + \ldots + \frac{x^4}{4!}\right)\right] +$$

$$+ \frac{5}{64}\left[1 - \exp(-x)\left(1 + \frac{x}{1!} + \ldots + \frac{x^6}{6!}\right)\right]. \qquad (7)$$

The structure of the series is such that in each successive approximation one term is added and the ones already found are unaltered. Hence, the third and second approximations can be found from (7) by discarding the last, or last two, expressions in the square brackets.

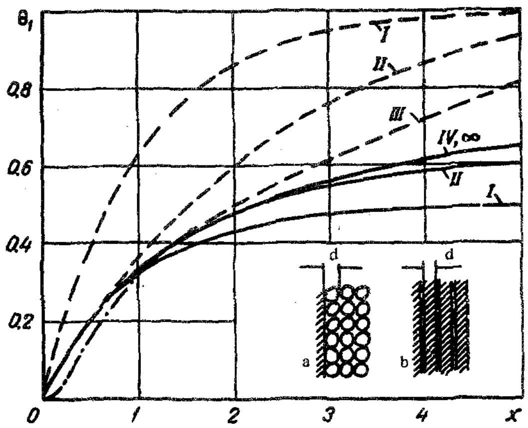

Fig. 1. Dimensionless temperature Θ_1 of the first row of particles in contact with the surface as a function of the dimensionless time τ (the dimensionless complex x is plotted on the x-axis). The equivalent system (b) replacing the actual regular arrangement of the particles (a) is shown on the right. The solid lines give the approximation "from below" [Eq. (7)] and the dashed lines give the approximation "from above" [Eq. (8)]. The figures on the curves denote the number of the approximation. The dot-dash line represents $\Theta_1 = \mathrm{erfc}\,\frac{1}{\sqrt{2x}}$ [Eq. (9)].

In the figure the first, second, and fourth approximations are represented by solid lines. It will be shown below that further approximations are unnecessary.

It is often assumed [2, 6] that when the time of contact of the packet with the surface is short all the heat is absorbed by the first row of particles and the presence of the second row does not affect the process. This is actually equivalent to the assumption that there is perfect heat insulation between these rows. Then

$$\Theta_1^{\mathrm{I}} = 1 - \exp(-x/2).$$

It is more suitable as a first approximation "from above" to use x rather than x/2 for the exponent. Then, for the last layer

$$\Theta_i = \exp(-x)\int_0^x \exp(x)\,\Theta_{i-1}\,dx.$$

Table 2

Values of Dimensionless Instantaneous Heat Transfer Coefficient $\alpha_\tau d/\lambda_{c.p}$ from Different Formulas

From formula	Values of $\alpha_\tau d/\lambda_{c.p}$ for x equal to							
	0	0.2	0.4	0.6	1.0	1.5	3.0	50
$\sqrt{c_m \rho_{c.p} d^2/\pi\tau\lambda_{c.p}}$	∞	1.78	1.26	1.03	0.8	0.65	0.46	0.11
$\alpha_\tau d/\lambda_{c.p} = 1 - \theta_1$	1	0.91	0.835	0.775	0.673	0.585	0.44	0.11
for θ_1 from (5')								
from (10)	1	0.97	0.88	0.8	0.69	0.58	0.44	0.11
from (11)	1	0.91	0.83	0.77	0.72	0.61	0.52	0.5
from (1')	2	1.1	0.91	0.8	0.69	0.58	0.44	0.11
$R_\lambda/R_{con} = \sqrt{2\pi x}$	0	1.12	1.58	1.94	2.5	3.06	4.34	17.7

For all the other layers below the i-th the temperature, as before, is given by (5), since the layers make thermal contact on both sides. By a method similar to the previous one of successive approximations we obtain a series of values of Θ_1. For instance, in the fifth approximation (further ones are usually unnecessary)

$$\Theta_1^V = 1 - \exp(-x) - \exp(-x) \times$$
$$\times \left\{ \frac{1}{2}\left[\frac{x}{1!} + \frac{x^2}{2!}\right] + \frac{1}{2^2}\left(1 + \frac{1}{2}\right) \times \right.$$
$$\times \left[\frac{x^3}{3!} + \frac{x^4}{4!}\right] + \frac{1}{2^2}\left(1 + \frac{1}{2^2}\right)\left[\frac{x^5}{5!} + \frac{x^6}{6!}\right] +$$
$$\left. + \frac{1}{2^2}\left(1 + \frac{1}{2^4} + \frac{1}{2^5}\right)\left[\frac{x^7}{7!} + \frac{x^8}{8!}\right]\right\}. \qquad (8)$$

As before, each previous approximation can be obtained from (8) by successively discarding the last term of the sum in the braces. The first three approximations calculated by this method are shown by dashed lines in the figure.

The figure shows that calculations from (7) and (8) give the same results, beginning with the fourth approximation (in the range of values of x of interest for a fluidized bed). When $x < 0.5$ the solution is given almost exactly by the first approximation "from below" [Eq. (6)], and when x is large ($x > 1$) the solution is given by an equation giving the change in temperature with time at a distance d from the surface of the body, if the multilayer system is regarded as a homogeneous medium with parameters c_m, $\rho_{c.p}$, and $\lambda_{c.p} = d/r_{con}$ (this is easily obtained from the known solution of the problem [5] of heating of a homogeneous mass):

$$\Theta_1 = \operatorname{erfc}\frac{1}{\sqrt{2x}}. \qquad (9)$$

The instantaneous value of the heat transfer coefficient is $\alpha_\tau = (1 - \Theta_1)r_{con}^{-1}$. The difference $(1 - \Theta_1)$ found from (9) for $x < 1$ also differs slightly from the value found from (8) or (7) (compare the solid and dot-dash lines in the figure for $x < 1$). Hence, for the calculation of Θ_1, which is contained in α_τ, Eq. (9) can be used in the whole temperature range. Using (9) and expanding the complex x contained in it, we obtain

$$\frac{\alpha_\tau d}{\lambda_{c.\phi}} \approx \operatorname{erf}\sqrt{\frac{c_m \rho_{c.p} d^2}{4\tau\lambda_{c.p}}} = \operatorname{erf}\sqrt{\frac{1}{2x}}. \qquad (10)$$

When x is small (< 0.5) a more accurate value of Θ_1 is given by Eq. (6) (see Fig. 1) and, hence, α_τ, obtained from (6), is also more accurate:

$$\frac{\alpha_\tau d}{\lambda_{c.p}} = \frac{1}{2}\left[1 + \exp(-x)\right]. \qquad (11)$$

It is of interest to note that this quantity differs from the corresponding expressions obtained in [2, 6] for the case where the temperature of the second row during the time of contact of the packet with the surface is unaltered. This means that even with such conditions it is necessary to take into account the heat transfer from the first row of particles to the second on the assumption that the temperature of the second is equal to the initial temperature of the packet.

In [4] it was assumed in the deduction of the formula for R_{con} that $R_{con} = d/2\lambda_{c.p} = r_{con}/2$. If we substitute this expression for R_{con} in (1), then formula (1) will take the form

$$\alpha_\tau d/\lambda_{c.p} = 2 \exp(2x)\operatorname{erfc}\sqrt{2x}. \qquad (1')$$

Table 2 compares the results of calculation from the different formulas. This table also gives the values of R_λ/R_{con} on the assumption that $R_{con} = 0.5r_{con}$.

The table shows that formula (1') gives a satisfactory approximation for $x \geq (0.5\text{–}1)$, i.e., for $R_\lambda/R_{con} > 2\text{–}2.5$. For these values the "continuous" phase of a fluidized bed can be regarded as a homogeneous medium, and the heat transfer can be calculated from formula (3). When $x < 0.5$ the "continuous" phase must be regarded as a discrete medium and Eq. (11) should be used for the calculation.

NOTATION

α_τ, α', α are the instantaneous, mean (during time of contact of packet), and effective (allowing for time of contact with bubble) heat transfer coefficients; β is the correction factor ($\beta = 1\text{–}1.2$); $\lambda_{c.p}$ and $\rho_{c.p}$ are the thermal conductivity and density of continuous phase of the fluidized bed; φ_t is the frequency of change of packets of particles at the surface; τ and τ_c are the variable time and time of contact of the packet with the surface; Θ_i^j is the dimensionless excess temperature of i-th layer in j-th approximation; A is the dimensionless coefficient [formula (2')]; c_m and d are the specific heat and diameter of particles of the material; f_0 is the fraction of time during which the surface is in contact with gas bubbles; R_λ and R_c are the thermal resistances of the packet at instants τ and τ_c; R_{con} is the contact thermal resistance; r_{con} is the thermal resistance between rows of particles; t_0, t_i are the initial and instantaneous temperatures of i-th layer of particles; t_w is the temperature of surface; x and y are the dimensionless complexes [formulas (2') and (4')]; Fo is the Fourier number.

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